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SUPPLEMENTAL DATA AND CALCULATIONS OF THE LATERAL STABILITY OF AIRPLANES

By Gotthold Mathias

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In connection with the DVL (Deutsche Versuchsanstalt für Luftfahrt) Report 272 (reference 1) on the theory of the lateral stability of airplanes, the formal results are here amplified in some respects and their technical significance again briefly explained.** Three numerical examples show how model tests for checking the lateral stability are to be evaluated and supplemented, if necessary, and how the stability limits depend on the design of the airplane and on the conditions of flight.

I. ASSUMPTIONS AND LIMITS OF LATERAL-STABILITY CALCULATIONS

The DVL Report 272 (reference 1) affords a general survey of the theory of the lateral stability of airplanes for the purpose of giving airplane designers suggestions for the technical fulfillment of stability requirements. In its essential features the investigation follows the lines well known in Germany, especially as given in the textbook of Fuchs and Hopf. In numerous details, however, the adaptation to the flight requirements of the present necessitated deviations, especially as regards evaluation of the order of magnitude, refinement or disregard of individual terms in the equations of motion.

The customary division of the general motion of an airplane into the symmetrical longitudinal motion and the unsymmetrical lateral motion assumes, on the one hand, the

*"Ergänzungen und Berechnungsbeispiele zur Seitenstabilitätslehre." Z.F.M., October 14, 1933, pp. 527-29, and October 28, 1933, pp. 563-68.
**The systems of coordinates, notation, and numbering of the formulas are the same here as in the earlier report (reference 1).

disregard of the influence of variations in the lateral angle and in the rolling and turning velocities on the dynamics of the longitudinal motion, and, on the other hand, the disregard of the influence of the variations in the angle of attack, in the forward speed and in the tilting velocity on the dynamics of the lateral motion, as likewise the disregard of all gyroscopic effects. Calculation and experience show that these conditions are generally better fulfilled for the longitudinal motion than for the lateral motion. Thus the results of a special investigation of the lateral motion yield a lower degree of mathematical reliability than the very satisfactory results for the purely longitudinal motion as confirmed by experience.

The existing knowledge regarding the calculation of the static longitudinal stability suffices, with some reliability, to control the here determining influences and to determine in advance the limiting values for the location of the center of gravity or the size of the horizontal tail surfaces. As regards the calculation of the lateral stability, however, we yet have practically no knowledge from experience. There are, moreover, very few available data from model and flight tests and these are mostly of no use for general application. For these reasons numerical considerations regarding lateral stability can yield, for the time being, only an estimation of the limits and indications of the methods to be adopted for improving the unsatisfactory stability conditions. Any preliminary calculation of the lateral stability, as in the case of longitudinal stability, is therefore possible only when sufficient data from model tests are available. As to the applicability of test results to other structural forms not exactly similar to the models, knowledge can be gained from experience only in the course of time.

In this report the results of the theory of lateral stability will be supplemented as regards the influence of the absolute airplane dimensions. The technical results of the theory will then be applied to three numerical examples for the purpose of showing the way to the practical evaluation of the data obtained.

II. STABILITY CONDITIONS AND THEIR DEPENDENCE ON THE ABSOLUTE AIRPLANE DIMENSIONS

In the three lateral-motion equations, the forces and moments can be represented in the form of simple time factors (see table I of DVL Report 272, reference 1), whose principal components (in brackets) consist of nondimensional coefficients and dimensional ratios. Nevertheless, the influence of the initial flight condition and dimensional ratios of the airplane on the course of the disturbing motion are not yet clearly manifest, since the mutually dependent quantities v, q, G/F and c_a (or c_n in the factor k_{ω_y}) occur together. The relation between these four quantities is given by the expressions

$$v = \sqrt{\frac{G \cos \varphi_0}{\rho/2 \; F \; c_a}} = v_e \sqrt{\frac{\cos \varphi_0}{c_a} \frac{\rho_0}{\rho}}$$

$$\frac{q}{G/F} = \frac{\cos \varphi_0}{c_a}$$

Hereby the laws of motion determined by the time factors can be referred to the independent quantities v_e (or G/F), ρ/ρ_0, c_a and $\cos \varphi_0$. In the time factors we must put

$$\frac{g}{v} \frac{q}{G/F} = \frac{g}{v_e} \sqrt{\frac{\cos \varphi_0}{c_a} \frac{\rho}{\rho_0}}$$

$$\frac{g}{b} \frac{q}{G/F} = \left(\frac{g}{v_e}\right)^2 \frac{\cos \varphi_0}{c_a} \frac{\rho}{\rho_0} \frac{G/g}{\rho \; F \; b/2}$$

The influences of the air density are expressed as independent factors in developed form with the aid of the "unit velocity"

$$v_e = \sqrt{\frac{2}{\rho_0} \frac{G}{F}}$$

which depends exclusively on the wing loading of the airplane (contrary to the previous representations in German

and foreign reports. The ratio $\frac{G/g}{\rho\ F\ b/2}$ can be designated, according to Glauert's method (reference 2), as the relative density of the airplane. This quantity represents a relation between the mass of the airplane and its dimensions by taking as the criterion the mass of an air prism over the wing area of the height of half the wing span.

Even the lateral-force component of the airplane weight due to the lateral inclination $(Z_\mu = G \cos \varphi_o\ \mu)$, which, after division by the momentum $v(G/g)$, appears in the form $(g/v) \cos \varphi_o\ \mu$, can be transformed according to the unitary structure of the other time factors. It is

$$\frac{g}{v} \cos \varphi_o = \frac{g}{v_e}\sqrt{\frac{\cos \varphi_o}{c_a}\ \frac{\rho}{\rho_o}\ c_a}$$

The quantity $\frac{g}{v_e}\sqrt{\frac{\cos \varphi_o}{c_a}\ \frac{\rho}{\rho_o}}$ is common to all the time factors. Its reciprocal value represents a time unit T dependent on the wing loading (v_e), the initial condition $(\cos \varphi_o/c_a)$ and the air density (ρ/ρ_o)*. Introduction of the time unit

$$(T^{-1}) = \frac{g}{v_e}\sqrt{\frac{\cos \varphi_o}{c_a}\ \frac{\rho}{\rho_o}}\ [s^{-1}]$$

makes the phenomena in the disturbing motions directly comparable for different airplanes.

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*For $\frac{\cos \varphi_o}{c_a} = 1$ and $\frac{\rho}{\rho_o} = 1$ this time unit (v_e/g) is the acceleration period from the velocity 0 to v_e in free fall under the influence of gravity.

The time factors for the forces and moments accordingly receive the following forms.

$$z_{\mu} = (T^{-1}) \qquad [c_a] \qquad [s^{-1}]$$

$$z_{\tau} = (T^{-1}) \qquad \left[-c'_{q_0}\frac{F_o}{F} + c'_{n_s}\frac{F_s}{F} + c_w\right] \qquad [s^{-1}]$$

$$l_{\tau} = (T^{-2}) \frac{G}{g\ \rho\ F\ b/2}\left(\frac{b}{i_y}\right)^2\left[c'_{m_{s_0}} + c'_{n_s}\frac{F_s}{F}\frac{l_s}{b}\right] \qquad [s^{-2}]$$

$$l_{\omega_y} = (T^{-1}) \qquad \left(\frac{b}{i_y}\right)^2\left[c_{d_{s_0}} + c'_{n_s}\frac{F_s}{F}\left(\frac{l_s}{b}\right)^2\right] \qquad [s^{-1}]$$

$$l_{\omega_x} = (T^{-1}) \qquad \left(\frac{b}{i_y}\right)^2\left[\frac{1}{12}\ c_{(t)}\right] \qquad [s^{-1}]$$

$$k_{\tau} = (T^{-1}) \frac{G}{g\ \rho\ F\ b/2}\left(\frac{b}{i_x}\right)^2\left[c'_{m_{q_0}} + \frac{1}{2}\frac{c'_{a_\infty}}{1 + \frac{2\ c'_{a_\infty}}{\pi\ \Lambda}}\frac{s_u}{b/2}\ \upsilon\right] \qquad [s^{-2}]$$

$$k_{\omega_y} = (T^{-1}) \qquad \left(\frac{b}{i_x}\right)^2\left[\frac{c_n}{2}\left(\frac{i_n}{b/2}\right)^2\right] \qquad [s^{-1}]$$

$$k_{\omega_x} = (T^{-1}) \qquad \left(\frac{b}{i_x}\right)^2\left[\frac{1}{12}\frac{c'_{a_\infty}}{1 + \frac{2\ c'_{a_\infty}}{\pi\ \Lambda}}\right] \qquad [s^{-1}]$$

(See table I in reference 1.)

The deciding coefficients for the stability of the disturbing motion (differing somewhat from the form in the previous report) are:

$$\left.\begin{aligned}
B &= (k_{\omega_x} + l_{\omega_y}) + z_T \\
C &= (k_{\omega_x}\, l_{\omega_y} - k_{\omega_y}\, l_{\omega_x}) + l_T + z_T\, (k_{\omega_x} + l_{\omega_y}) \\
D &= (k_{\omega_x}\, l_T - k_T\, l_{\omega_x}) + z_\mu\, (k_T - l_T \tan\varphi_0) + \\
&\qquad + z_T\, (k_{\omega_x}\, l_{\omega_y} - k_{\omega_y}\, l_{\omega_x}) \\
E &= z_\mu [\, (k_T l_{\omega_y} - k_{\omega_y}\, l_T) - (k_{\omega_x}\, l_T - k_T\, l_{\omega_x}) \tan\varphi_0 \,]
\end{aligned}\right\} \qquad (21)$$

Technically important for the dynamic and static directional stability of normal flight is only the double condition

$$\frac{C}{B}\, D > E > 0, \qquad (22)$$

in which B and C can always be assumed to be positive.

The time factors and the stability coefficients derived from them consist chiefly of nondimensional coefficients which vary little or not at all with the geometrical dimensions of the airplane. The aerodynamic coefficients are the same for all geometrically similar structures, and their variation for differing structures is mostly within narrow limits. Even the relations of the radii of gyration to the wing span are subject to only slight variations for the same general airplane structure. Great variations are due almost exclusively to differences in the power plants (one or more engines, mounted low or high).

The only direct dependence on the airplane dimensions appears in the two time factors of the static stability about the vertical and longitudinal axes (l_T and k_T), in which the "relative density" of the airplane $\dfrac{G}{g\,\rho\,F\,b/2}$ occurs. This quantity (aside from the air density and wing loading) depends on the span, i.e., on an absolute dimension of the airplane. This dependence is almost meaningless for the dynamic stability conditions, since, according to equation (21), the relative density (from l_T or k_T) does not occur at all in B; is mostly subordinate in C; and appears in D, E, and R ($\approx B(CD - BE)$) as a common factor in the main portions. It is nevertheless of great

importance for the course of the individual components of a disturbing motion with respect to time.

The time unit T is likewise important only when the course of the motion itself is to be followed under different assumptions. Aside from the air density and initial condition, it depends only on the unit velocity v_e and wing loading, which varies within the same limits for large and small airplanes. The time unit therefore has no influence on the stability conditions.

III. TECHNICAL DEDUCTIONS FROM THE STABILITY CONDITIONS

In sections III and IV of the previous report (reference 1), the stability conditions contained in the double inequality (22) are thoroughly discussed. From the standpoint of the airplane designer, the static lateral stability $(E > 0)$ requires particular attention. The dynamic condition $(C/B)D > E$, on the other hand, is shown to be easily fulfilled by positive directional stability. The structural means for attaining static lateral stability are only briefly presented here according to the results of the deductions in the previous report (reference 1), since they form the basis for the interpretation of the following numerical examples.

Portions of the wing and fuselage (including wheels, floats, engine nacelles, etc.) play a prominent part in the "turn damping" and especially in the static directional stability. With respect to the rudder effect, the "relative turn damping" is defined by

$$\delta = \frac{c_{d_{s_0}} F b^2 + c'_{n_s} F_s l_s^2}{c'_{n_s} F_s l_s^2} = 1 + \frac{c_{d_{s_0}} F b^2}{c'_{n_s} F_s l_s^2} \qquad (24)$$

and the "relative directional stability" by

$$\sigma = \frac{c'_{m_s} F b + c'_{n_s} F_s l_s}{c'_{n_s} F_s l_s} = 1 + \frac{c'_{m_{s_0}} F b}{c'_{n_s} F_s l_s} \qquad (25)$$

The relative turn damping cannot be greatly influenced by structural means. For the most part it has an order of magnitude of 1.2 to 1.4. The relative directional stability

on the contrary, is very sensitive to structural measures, such as the arrangment of the wheels, floats, length of engine nacelles, height and shape of fuselage and, above all, the location of the center of gravity of the airplane with respect to the length of the fuselage. The influence of the wing is relatively small. An inherently unstable fuselage effect in the directional stability $(\sigma < 1)$ is found to be favorable for the lateral stability. The size of the vertical tail surfaces should be sufficient for directional stability and lateral control, but, for reasons of safety, should not be excessive. A great length of tail is advantageous for the static lateral stability, only when the unstable effect of the fuselage on the directional stability is relatively small $(\sigma > \delta/2)$ or when, with an increase in the length of the tail, the directional stability can be maintained with a reduction in the area of the vertical tail surfaces, a constant location of the center of gravity and an unchanged shape of the front part of the fuselage being assumed.

The moments about the longitudinal axis of the airplane come almost exclusively from the wing. The rolling moment in turning should be kept as small as possible. The meeting of this requirement is facilitated by a moderate aspect ratio of the wing. The flattening of the lift distribution toward the wing tips by warping the wing is the most important means for damping the rolling moment in turning. Decisive for the lateral stability is the static transverse stability obtained by a suitable dihedral of the wing. By introducing equations (24) and (25), we obtain

$$\nu \geqq \frac{c_n}{c'_{a_\infty} f_M} \frac{\left(\frac{i_n}{b/2}\right)^2}{\frac{s_u}{b/2}} \frac{b}{l_s} \frac{\sigma}{\delta} - \frac{2\ c'_{m_{q_0}}}{c'_{a_\infty} f_M \frac{s_u}{b/2}} \tag{26a}$$

or, as an approximation for the now customary structural forms,

$$\nu \geqq \frac{\frac{2}{3} c_n \frac{b}{l_s} \frac{\sigma}{\delta} - 4\ c'_{m_{q_0}}}{c'_a \left[1 - \left(\frac{b_i}{b}\right)^2\right]} \tag{26b}$$

The qunatity $c'_{m_{q_0}}$ takes into account the influence of a

transverse stability of the whole airplane, already present even without the wing dihedral. This transverse stability sometimes reaches quite high values due to the sweepback of the wings, to the height of the vertical tail surfaces and to other structural characteristics.

Model tests by the customary six-component method lead to conclusions regarding the lateral-stability conditions, when made in accordance with the determining viewpoints. Unfortunately such has not generally been the case. It can easily be accomplished, however, as soon as the requirements are clearly understood. At least two series of tests are necessary: tests with the complete model and tests without the vertical tail surfaces. Moreover, a third series of tests with different wing dihedrals is useful, in case it is necessary to change the dihedral adopted in the design of the airplane. The details regarding the evaluation of such six-component tests are given in section V of the previous report. In short, the condition, according to which the tests enable conclusions regarding the lateral stability, can be expressed by the approximation formula

$$\frac{c'_{m_q}}{c'_{m_s}} \gtrapprox \frac{1}{8} \frac{c_n}{(c'_{m_s} - c'_{m_{s_0}}) \frac{l_s}{b}} \tag{30b}$$

If the model is not satisfactory according to the first test results, the requisite enlargement of the dihedral can be made according to equation (26a) or (26b). Here the quantity

$$\sigma = \frac{c'_{m_s}}{c'_{m_s} - c'_{m_{s_0}}}$$

obtained from the tests must be introduced for the relative directional stability, and the measured increase c'_{m_b} in the rolling moment for the transverse stability $c'_{m_{q_0}}$. If the results of model tests with different dihedrals are available, the dihedral needed for the desired degree of lateral stability is then easily ascertained.

IV. NUMERICAL EXAMPLES

1. Evaluation of the Tests of a Model of a High-Wing Sport Monoplane

The original purpose of these model tests, which determines their systematic performance, is chiefly the solution of control problems for all three axes. The most important values can be taken for the consideration of the lateral stability. There are lacking six-component tests without vertical tail surfaces, instead of which, however, the available three-component tests of the vertical tail surfaces alone, with horizontal tail surfaces and fuselage as a "screen", can be utilized. The likewise lacking rolling-moment tests with different wing dihedrals must be replaced by approximate calculations.

The following mean values must be taken directly from the model tests.

With complete model:

$$(c'_{m_q}) = 0.343, \quad (c'_{m_s}) = -0.264, \quad (c'_q) = -0.353.$$

With model having vertical tail surfaces: $\dfrac{\partial\, c_{a_s}}{\partial\, \alpha_s} = 1.1$

With model of airplane without horizontal tail surfaces: $c'_a = 4.35$.

The ascending coefficients refer to angles in circular measure. In the representation of the test results, the reference point for the moments is the leading edge in the middle of the wing. The coefficients c_{m_q} and c_{m_s} are based on F t; $c'_{m_s} < 0$ indicates stability.

The test results must first be converted to correspond to the c.g. of the airplane (rearward position r/t, upward position h/t, with respect to the reference point of the moments) and to the span of the wing.

a) Moments about the longitudinal axis.

$$c'_{m_q} = \left[(c'_{m_q}) + (c'_q)\ \frac{h}{t}\right] \frac{t}{b}$$

$$\frac{h}{t} \approx -0.36,\ \frac{t}{b} = 0.1328$$

$$c'_{m_q} = [0.343 + 0.353 \times 0.36]\ 0.1328 = 0.0625$$

For comparison the transverse-moment increase for the wing alone is estimated on the basis of the model test. According to equation (2) of the previous report (reference 1)

$$c'_{m_q} = \frac{c'_{a_\infty}\ f_M}{2}\ \frac{s_u}{b/2}\ \nu$$

where, instead of ν, the actual dihedral angle (2.5°) with respect to a backswept portion (estimated at about 1.5°), i.e. $\nu \approx 0.07$ rad., is introduced. The value $c'_{a_\infty}\ f_M$ can be derived from the value c'_a of the model test according to the formula*

$$c'_{a_\infty}\ f_M = \frac{c'_a\ \pi\ \Lambda}{c'_a + \pi \Lambda}$$

For $c'_a = 4.35$ and $\Lambda \approx 7.6$ we accordingly obtain the value $c'_{a_\infty}\ f_M = 3.68$. With $\frac{s_u}{b/2} \approx 0.5$ (approximately rectangular outline of the wing halves) we obtain

$$(c'_{m_q})_{wing} = \frac{3.68}{2}\ 0.5 \times 0.07 = 0.0644$$

This value differs but little from the previous one obtained from model tests. Moreover it leads to the conclusion that the influence of the fuselage, tail and landing gear is altogether negligible.

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*This formula can be derived from the expressions (accurately valid for elliptical wings) for the marginal effect

$$c'_a = \frac{c'_{a_\infty}}{1 + \frac{c'_{a_\infty}}{\pi \Lambda}} \quad \text{and} \quad (c'_{a_\infty}\ f_M) = \frac{c'_{a_\infty}}{1 + \frac{2\ c'_{a_\infty}}{\pi\ \Lambda}}$$

b) Moments about the vertical axis.

$$c'_{m_s} = \left[-(c'_{m_s}) + (c'_q)\,\frac{r}{t}\right]\frac{t}{b} \qquad (\text{Stability } c'_{m_s} > 0)$$

$$= 0.035 - 0.047\,\frac{r}{t}$$

According to the model sketches

$$\frac{F_s}{F} = 0.0882, \quad \frac{l_s}{b} \approx 0.36.$$

Consequently the vertical-tail-surface effect

$$\frac{\partial c_{a_s}}{\partial \alpha_s}\,\frac{F_s}{F}\,\frac{l_s}{b} = 1.1 \times 0.0882 \times 0.36 = 0.035$$

The point of application of the lateral aerodynamic forces to the fuselage accordingly lies directly under the moment reference point of the model test (leading edge in center of wing), since, for $r/t = 0$ according to the first formula, $c'_{m_s} = 0.035$, i.e., the same as the vertical-tail-surface effect.

c) Static lateral stability.

According to equation (30b)

$$\frac{c'_{m_q}}{c'_{m_s}} \underset{\approx}{>} \frac{1}{8}\,\frac{c_a}{\dfrac{\partial c_{a_s}}{\partial \alpha_s}\,\dfrac{F_s}{F}\left(\dfrac{l_s}{b}\right)^2}$$

c'_{m_s} depends on r/t. Hence a limiting line $c_a = \text{Fkt}(r/t)$ can be determined, separating the stable from the unstable regions (fig. 1). This line has the equation

$$\frac{r}{t} = \frac{0.035}{0.047} - \frac{8 \times 0.035 \times 0.36 \times 0.0325}{0.047\,c_a} = 0.745 - \frac{0.134}{c_a}$$

It is obvious that the airplane in the practical c.g. region $(0.35 < r/t < 0.45)$ is already laterally unstable in gliding flight at $c_a \approx 0.4$.

The extension of the lateral stability to higher lift coefficients is easily determined, when the variation in the transverse stability with the dihedral angle is known from model tests. In the present case, however, the necessary change in the dihedral can be determined only by calculation. The goal is reached in the simplest way by utilizing the linear relation between c_a and ν (equation 26). According to figure 1, the c_a limit is known for every c.g. location for the present dihedral of 2.5 percent. According to equation (26a), the theoretical limit $c_a = 0$ would be reached by the airplane with a change in the dihedral amounting to

$$(\Delta \nu)_o = \frac{-2\, c'_{m_q}}{c'_{a_\infty}\, f_M\, \frac{s_u}{b/2}}$$

With the numerical values already used above, we obtain

$$(\Delta \nu)_o = \frac{-2 \times 0.0625}{3.68 \times 0.5} \approx -3.9^o$$

The point thus obtained for $c_a = 0$ is common to all c.g. locations. Figure 2 represents the relation between the stability limits c_a and the dihedral of the wing for four c.g. locations. This leads to the conclusion that (e.g.), at $r/t = 0.4$ and a dihedral increased to 4.8^o, lateral stability is to be expected up to $c_a = 0.6$.

2. Evaluation of the Tests of a Model of a Low-Wing Training Airplane

The available results of model tests ought to inform us regarding the controllability about the three airplane axes in various angles of attack. They are of very little use for determining the lateral stability, because they contain no data regarding tests without vertical tail surfaces nor with vertical tail surfaces alone, nor regarding rolling-moment tests at different wing dihedrals. The evaluation therefore depends largely on mathematical esti-

mates, the test data indicating only the general order of magnitude. Despite these limitations, the evaluation affords us valuable information.

The model tests yield directly the following mean values of the derivatives in arc units:

$$(c'_{m_q}) \approx 0.14, \quad (c'_{m_s}) = -0.42 + 0.11\ c_a$$

$$(c'_q) = -0.44 + 0.07\ c_a$$

The quantities (c'_{m_s}) and (c'_q) are noticeably affected by the angle of attack, apparently due to low-wing arrangement. The representation in terms of c_a is made possible by the available results of the three-component tests with the wing and with the complete model. The lift increase of the wing $(\Lambda = 8.5)$ is $c'_a = 4.45$. The measured moment coefficients are based on the $F\ t_i$. The reference point for the moments is the front end of the chord in the middle of the wing. $c'_{m_s} < 0$ denotes stability.

The mathematical conversion with respect to the c.g. of the airplane and the wing span as reference data, according to the formulas given in the previous example, yields, for the moments about the longitudinal axis (with $h/t_i = 0.16$, $t_i/b = 0.14$), the value

$$c'_{m_q} = [0.14 + (-0.44 + 0.07\ c_a)\ 0.16]\ 0.140$$

$$= 0.01\ (+ 0.0016\ c_a)$$

Due to the small wing dihedral (about $1.5° = 0.026$ rad.) the amount of the rolling-moment increase is very sensitive to variations in the lateral forces. Comparison with the calculated wing effect can therefore yield no criterion regarding the accuracy of the calculation. The wing effect, according to the formula (see preceding example)

$$(c'_{m_q})\text{wing} = \frac{c'_{a\infty}\ f_M}{2}\ \frac{s_u}{b/2}\ \nu = \frac{1}{2}\ \frac{c'_a\ \pi\ \Lambda}{c'_a + \pi\ \Lambda}\ \frac{s_u}{b/2}\ \nu$$

(with $\frac{s_u}{b/2} = 0.45$) is

$$(c'_{m_q})\text{ wing} = \frac{3.8}{2} \times 0.45 \times 0.026 = 0.022.$$

This value (calculated with respect to b as the reference length) approximates the value of the model test

$$(c'_{m_q})\frac{t_i}{b} = 0.14 \times 0.140 \approx 0.02$$

which indicates the point of application of the lateral force at about the height of the wing chord on the fuselage (high landing gear included).

The calculation of the moments about the vertical axis yields

$$c'_{m_s} = [-(-0.42 + 0.11\ c_a) + (-0.44 + 0.07\ c_a)\tfrac{r}{t}]\ 0.140$$
$$= 0.059 - 0.015\ c_a - (0.062 - 0.010\ c_a)\ \frac{r}{t}.$$

These numbers indicate an exceptionally great directional stability. They show that neutral equilibrium ($c'_{m_s} = 0$) is obtained at the rearward c.g. locations

$$\frac{r}{t} = \frac{0.059 - 0.15\ c_a}{0.062 - 0.10\ c_a}$$

and indeed for

	$c_a =$	0.2	0.4	0.6	0.8
at	$\frac{r}{t} \approx$	0.93	0.91	0.89	0.87

The share of the vertical tail planes in the directional stability can, for lack of test data, be estimated only from other similar tests. It is approximately

$$\frac{\partial\ c_{a_s}}{\partial\ \alpha_s}\ \frac{F_s}{F}\ \frac{l_s}{b} \approx 1.5 \times 0.05 \times 0.32 = 0.024$$

On deducting this value from the measured directional stability, it is found that the airplane without vertical tail surfaces is in neutral equilibrium at the rearward c.g. location

$$\frac{r}{t} = \frac{0.035 - 0.15\ c_a}{0.062 - 0.10\ c_a}$$

For $c_a = 0.4$ this leads to $r/t = 0.5$ as the location of the center of pressure of the lateral force on the fuselage (short engine nacelle, great height of rear portion of fuselage). Accordingly the airplane without vertical tail surfaces is probably quite stable inherently with the c.g. locations ($r/t = 0.34$ to 0.40) occurring in practice. Its "relative directional stability" (equation 25) is

$$\sigma = \frac{0.059 - 0.015\ c_a}{0.024} - \frac{0.062 - 0.010\ c_a}{0.024}\ \frac{r}{t}$$

$$= (2.46 - 0.63\ c_a) - (2.58 - 0.42\ c_a)\ \frac{r}{t}$$

For $c_a = 0.4$ and $r/t = 0.37$ (by way of example) $\sigma \approx 1.32$, which is very unfavorable for obtaining static lateral stability.

The limit of the static lateral stability is again determined by the formula .

$$\frac{c'_{m_q}}{c'_{m_s}} \geqq \frac{1}{8}\ \frac{c_a}{\frac{\partial\ c_{a_s}}{\partial\ \alpha_s}\ \frac{F_s}{F}\ \left(\frac{l_s}{b}\right)^2}$$

With the preceding numerical values we have, as the equation of the limiting curve $r/t = f(c_a)$,

$$\frac{0.01 + 0.0016\ c_a}{0.059 - 0.015\ c_a - (0.062 - 0.010\ c_a)\ \frac{r}{t}} = \frac{1}{8}\ \frac{c_a}{0.024 \times 0.32}$$

$$\frac{r}{t} = \frac{0.059 - 0.015\ c_a}{0.062 - 0.010\ c_a} - \frac{0.000614 + 0.0001\ c_a}{(0.062 - 0.010\ c_a)\ c_a}$$

Comparison of this limiting curve (fig. 3) with the corresponding curve of the first example (fig. 1) shows that, on this airplane with practically perfect lateral stability, increasing the dihedral can have no great effect. Corresponding to the data used for figure 2, the

quantity $(\Delta \nu)_0$ is about -0.7°, and the c_a limit for the existing dihedral is about 0.02 for all c.g. locations according to figure 3. The thus-determined jet, for the efficacy of changes in the dihedral, flows so smoothly that, for practical reasons, the necessary dihedral angles are no longer feasible.

It is quite profitable to follow this example mathematically, although it is not very satisfactory in its numerical bases and although the results are valid only in their general order of magnitude. It combines all the conditions which are unfavorable to the attainment of static lateral stability, namely: low position of wing with small dihedral angle (no transverse stability), short engine nacelle, relatively high rear portion of fuselage (inherently stable fuselage), and relatively short tail in comparison with the wing span ($l_s/b \approx 0.32$). Practical experience in cruising and in flat gliding with this airplane type demonstrates the qualitative correctness of the calculated results. Nevertheless the lateral instability is not disagreeable, so long as the pilot is sure of his position with respect to the horizon and can immediately correct every deviation, either consciously or unconsciously, by the habitual small motions of the controls.

3. Stability Investigation of an Airplane of the Customary Size and Design

a) Preliminary Remarks

While, in the two preceding examples, a few available data from model tests were used to determine the static lateral stability of the airplane type under investigation, an example will now be calculated for checking all, even the dynamic, limiting conditions of the lateral stability. As the basis for this, numerical values are adopted, which, according to structural calculations and model tests for conventional airplane types, may be regarded as mean values, but of course cannot be generalized.

The only value which directly expresses the absolute size of the airplane is the span. Aside from this, the only criterion is the wing loading, while all other data are nondimensional ratios or coefficients. The significance

of this fact for the stability conditions was discussed at the end of section II. The calculations are thus greatly simplified. A further slight facilitation might be effected by eliminating the time scale (T^{-1}), since this does not affect the stability conditions. This will be retained, however, in the following calculations.

b) Time Factors and Stability Limits for $c_a = 0.7$

The numerical calculations are based on the following characteristic values of the airplane

$$b = 20 \text{ m}, \; b/t_m = \Lambda = 8, \; G/F = 60 \text{ kg/m}^2,$$

$$(v_e = 31.0 \text{ m/s}),$$

$$\frac{i_x}{b} = \frac{1}{9}, \; \frac{i_y}{b} = \frac{1}{7}, \; \frac{l_s}{b} = 0.4,$$

$$c_w = 0.07 \text{ (for } c_a = 0.7), \; \cos \varphi_o \approx 1, \; (\sin \varphi_o \approx 0),$$

$$\frac{\rho}{\rho_o} = 0.96,$$

$$c'_{a\infty} f_M = 3.75, \; c'_{(t)} = -0.4; \; \frac{s_n}{b/2} = 0.47, \; \left(\frac{i_n}{b/2}\right)^2 = 0.30,$$

$$c_{d_{so}} = 0.004, \; c'_{n_s} = 2.0, \; c'_{qo} = -0.6, \; c'_{mqo} = 0.01,$$

$$c'_{m_{so}} = -0.01 - 0.6 \; r/b = -0.01 - 0.075 \; r/t_m.$$

(All coefficient gradients are in circular measure.)

The "relative density" of the airplane is accordingly

$$\frac{G}{g \; \rho \; F \; b/2} = \frac{60}{9.81 \times 0.120 \times 10} = 5.10,$$

the time scale*

$$(T^{-1}) = \frac{9.81}{31.0} \sqrt{\frac{1}{0.7} \times 0.96} = 0.370 \text{ s}^{-1}.$$

* See footnote on next page.

From this the following time factors (with $F_s/F \equiv f_s$) are calculated:

$$z_\mu = 0.259,$$

$$z_T = 0.248 + 0.74\ f_s,$$

$$l_T = -0.342 - 2.57\ r/t_m + 27.4\ f_s,$$

$$l_{\omega_y} = 0.073 + 5.80\ f_s,$$

$$l_{\omega_x} = -0.6,$$

$$k_T = 0.57 + 49.8\ \nu,$$

$$k_{\omega_y} = 3.15,$$

$$k_{\omega_x} = 9.36.$$

The coefficients of the principal equations are

$$B = 9.68 + 6.54\ f_s,$$

$$C = 4.57 - 2.58\ \frac{r}{t_m} + 90.2\ f_s,$$

$$D = -2.1 - 24.1\ \frac{r}{t_m} + 272\ f_s + 40.2\ {f_s}^2 + 42.8\ \nu,$$

---

*In order to avoid misunderstandings in comparing with previous works, especially with the numerical example of Fuchs and von Baranoff (reference 3), attention is here called to the fact that the time scale used by Fuchs and Hopf is otherwise defined

$$\left(\frac{v}{b} \text{ instead of } \frac{g}{v_e} \sqrt{\frac{\cos\ \varphi_o}{c_a}\ \frac{\rho}{\rho_o}} \right)$$

and therefore differs, even in the order of magnitude, from that here chosen (1.89 s^{-1} instead of 0.37 s^{-1}). The time scale of Fuchs and Hopf is not only a function of the wing loading, initial condition and air density, but also of the absolute airplane size (wing span). It is equal to the scale T^{-1} multiplied by the relative density of the airplane.

$$E = 0.259 \left[\left\{ (0.0114 + \nu)(3.6 + 289 f_s) - \left(-1.07 - 8.1 \frac{r}{t_m} + 86.4 f_s\right) \right\} - \left\{ \left(-2.84 - 24.1 \frac{r}{t_m} + 257 f_s\right) + 29.9 \nu \right\} \tan \varphi_o \right].$$

The fulfillment of the condition that, in stability, all coefficients must be positive, depends, with a given c_a, on three variables: r/t_m, f_s, and ν (fig. 4). $B > 0$ and $C > 0$ are always fulfilled, since f_s must always be greater than 0. $D > 0$ is fulfilled when

$$\nu \geqq 0.049 + 0.563 \frac{r}{t} - 6.35 f_s - 0.940 f_s^2 .$$

This limit is practically rectilinear, since, for all technically possible values of f_s, the summand $0.94 f_s^2 \ll 6.35 f_s$. $E > 0$ is fulfilled when, for the most important special case, horizontal flight, the relation

$$\nu \geqq \frac{-1.08 - 8.10 \frac{r}{t} + 86.4 f_s}{3.6 + 289 f_s} - 0.0114$$

is fulfilled. This condition is decisive for the dihedral.

$R = BCD - D^2 - B^2E \approx B(CD - BE) > 0$ yields a further limiting relation between ν and f_s. In horiztonal flight this must be

$$\nu \geqq \frac{12.4 + 125.1 \frac{r}{t_m} - 62.2 \left(\frac{r}{t_m}\right)^2}{187 - 110.4 \frac{r}{t_m} + 3129 f_s - 490 f_s^2} - \frac{(1260 - 2890 \frac{r}{t_m}) f_s - 24770 f_s^2 - 3630 f_s^3}{137 - 110.4 \frac{r}{t_m} + 3129 f_s - 490 f_s^2}$$

This limit (similar to $D = 0$) shows a practically perfect rectilinear course. It represents a very flat arc of the exact limit $R = BCD - D^2 - B^2E = 0$, which continues similarly to a hyperbola far beyond the practically possible region (fig. 5).

The inclination of the two limiting lines $D = 0$ and $R = 0$ is approximately determined by the relations

$$\frac{l_T}{k_T} \gtrapprox \frac{l_{\omega_x} - z_\mu}{k_{\omega_x}} \quad \text{for } D = 0$$

$$\frac{l_T}{k_T} \gtrapprox \frac{l_{\omega_x}}{k_{\omega_x}} \quad \text{for } R = 0$$

where l_T is a linear function of f_s and where k_T is a linear function of ν. The sign of the numerical value of l_{ω_x} (almost always negative) is decisive. Furthermore, it is apparent that the limit $D = 0$ of the stable region is always beyond the limit $R = 0$ and is therefore without practical importance. Since $R = 0$ for $D = 0$ and $E = 0$, these three limiting lines must intersect one another at the same point*.

The dynamically stable region is accordingly included between the limiting lines $E = 0$ and $R = 0$. It is worthy of note that dynamic lateral stability is possible, even with static directional instability, and also with a slight transverse instability. The main reason for this, as already metioned, is the generally negative roll-yawing moment l_{ω_x}. This stability region, however, is not utilizable in flight, since it expresses itself, when inclined, in reeling vibrations (stable at first but only slightly damped), as described by Gehlen (reference 4) and in my previous report (reference 1). The condition of static directional stability $l_T \geqq 0$ is the practically simplest and always sufficient stability limit in place of $R = 0$. It is independent of ν and therefore represents a parallel to the ν-axis with the abscissa

$$f_s = 0.0125 + 0.094 \frac{r}{t}$$

All technically important considerations are hereby limited between static directional indifference and static lateral indifference.

The effect of the inclination of the flight path on the static lateral stability can here be represented only on the express assumption that the stability conditions are not seriously affected by the propeller slipstream. A gliding flight ($\varphi_0 = \arctan c_w/c_a$) with zero propeller thrust can, of course, not be directly compared with a horizontal lfight ($\varphi_0 = 0$ with like c_a) with propeller

*Figure 1 in R. & M. 1077 (Garner, "Lateral Stability ...") is in this respect open to objection, even as a rough approximation.

thrust equal to the drag, without investigating the variations in the influence of the slipstream between these two operating conditions. The most important effect is to be expected on the directional stability, since the vertical tail surfaces are mostly in the slipstream. Exhaustive tests have not yet been made, however. Occasional tests show no consistent behavior. The addition or subtraction of the directional stability in powered flight, as compared with gliding flight, seems to be independent of the design of the airplane.

Notwithstanding this uncertainty, the limit of the static lateral stability in gliding flight is represented with zero propeller thrust, in order to show at least the order of magnitude of the effect of the inclination of the flight path. Hereby it is assumed that the disappearance of the slipstream influence is negligible. With $c_w = 0.07$ at $c_a = 0.7$, i.e. $\tan \varphi_o = -0.1$ or $\varphi_o \approx -6^o$, the equation of the limit $E = 0$ reads

$$\nu = \frac{-0.827 - 5.7 \frac{r}{t} + 57.4 f_s}{6.6 + 289 f_s}$$

The consequent requisite dihedrals are considerably smaller than in horizontal flight (fig. 4). It is therefore to be expected that an airplane which has only slight lateral stability in horizontal flight, will (with the same lift coefficient) show a noticeable increase in static lateral stability in gliding flight.

c) Lateral Stability in Terms of the Lift Coefficient c_a

Lateral disturbing motions in straightaway flight with different initial conditions (c_a) are distringuished at first by their dying away with time, which is designated by the time scale ($T^{-1} \sim c_a^{-\frac{1}{2}}$) in terms of c_a. As already mentioned, this variation has no effect on the stability conditions.

Beyond this, however, the time factors z_μ and k_{ω_y} are linearly dependent on c_a. The intermediate variability of the other time factors, whose aerodynamic coefficents according to model tests are sometimes slightly dependent

on c_a, is generally of subordinate importance in the middle c_a region (up to $c_a \approx 0.8$) and may here be disregarded. The stability condition $B > 0$ is scarcely affected in the normal region by variations in c_a. The condition $C > 0$ must never be endangered, since, due to $l_{\omega_x} \lesssim 0$, the summand $-k_{\omega_y} \, l_{\omega_x} \gtrsim 0$ must be regarded. The same result is reached by corresponding considerations regarding $D > 0$ and $R > 0$.

On the other hand the condition $E > 0$ of the static lateral stability is greatly influenced by changes in the initial magnitude of c_a. If the function

$$k_{\omega_y} = 4.50 \; c_a \left(= \frac{3.15}{0.7} \; c_a\right)$$

introduced instead of the value $k_{\omega_y} = 3.15$, which is valid for $c_a = 0.7$, the equation for the limiting lines $E = 0$ then reads:

$$\nu \approx \frac{-1.54 - 11.56 \frac{r}{t} + 123.5 \; f_s}{3.6 + 289 \; f_s} \; c_a - 0.0114.$$

In figure 6, c_a is introduced into the expression $\nu(f_s)$ as a parameter with the same c.g. location $r/t = 0.35$. For other c.g. locations, the corresponding groups of lines follow exactly the same course as the original displacement shown in figure 4.

Moreover, the limit $E = 0$ can be variously expressed in the form

$$(0.0114 + \nu)(3.6 + 289 \; f_s) - \\ - \left(-1.54 - 11.56 \frac{r}{t} + 123.5 \; f_s\right) c_a = 0$$

according to the choice of the parameter and axes (from the four variables ν, f_s, r/t, c_a), so that data can be obtained in a predetermined direction.

Of the six possible methods of presentation (aside from the likewise corresponding figure 6) only the two most

important ones will be considered.

In figures 7a to 7c three sizes of the vertical tail surfaces (F_s/F) and three dihedral angles (ν) are taken as parameters. The curves show up to what initial conditions (c_a), dependent on the rearward location of the c.g. (r/t), static lateral stability is to be expected. This manner of presenting the problem is the only one open for a fixed airplane form. Its graphic representation corresponds to the results of the two preceding examples according to the results of the model tests (figs. 1 and 3).

Figures 8a to 8c show, for three sizes (F_s/F) of the vertical tail surfaces and three c.g. locations (r/t), up to what initial conditions (c_a), dependent on the dihedral angle (ν), static lateral stability is present. This graph corresponds to the first numerical example (fig. 2).

Figures 6 to 8 show that, in the present case, it is possible, without unusual measures, to attain lateral stability within the region of normal lift.

V. SUMMARY

The DVL Report 272 on lateral stability (in addition to the formal discussion of the problem) explained the technical significance of the theoretical results and gave general suggestions for their practical evaluation in airplane design. In the sections I and II of this supplementary report a few additional data are introduced which are of importance for the mathematical application of the theory. In section III the previous technical results are again briefly summarized.

Section IV of this report contains three mathematical examples, two of which are evaluations of available model-test results and one a carefully elaborated numerical example. The evaluations are intended to show the minimum data necessary for a rough calculation of the lateral stability and how far incomplete data can be supplemented by additional estimations. The numerical example shows that, in addition to the static directional stability, only the static lateral stability of the airplane needs to be determined. This is affected, among other things, by the size of the vertical tail surfaces, the dihedral angle of the wing, by the rearward location of the c.g. and by the flight condition (c_a). The nature of these relations is

clarified by a series of graphs with various arrangements of parameters and axis variables.

The results show that, with a skillful utilization of all the possibilities in designing, lateral stability can always be attained insofar as considered necessary for the purpose of the airplane.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

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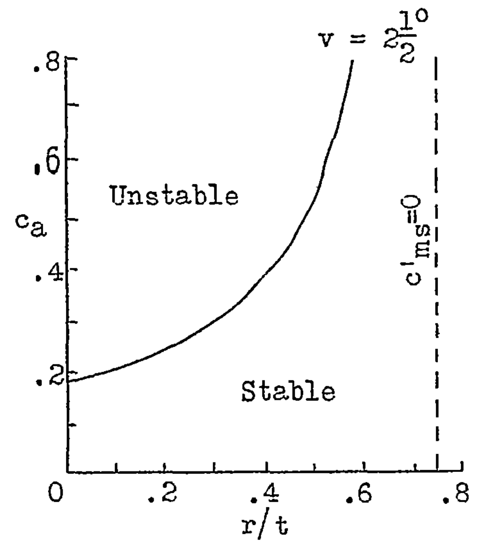

Figure 1.(example 1).-Limit of static lateral stability with given dihedral, in terms of c.g. location (r/t) and initial condition (c_a).

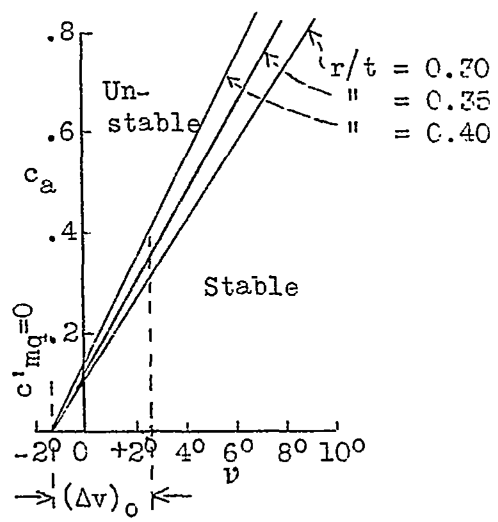

Figure 2.(example 1).-Limits of static lateral stability for three c.g. locations (r/t), in terms of the dihedral (ν) and initial condition (c_a).

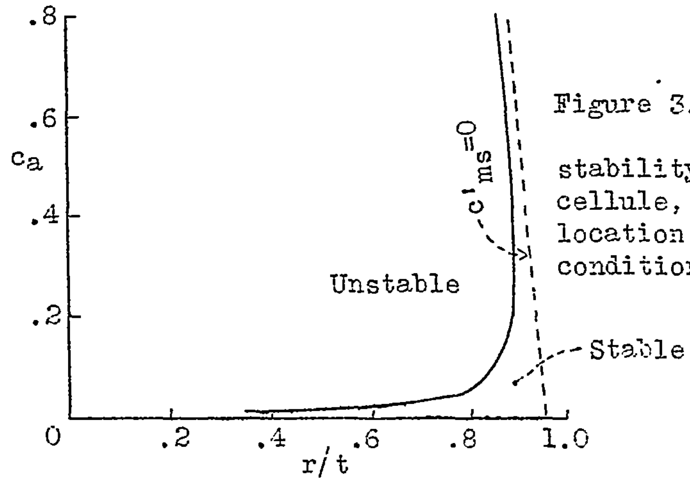

Figure 3.(example 2).-Limit of static lateral stability with given form of cellule, in terms of c.g. location (r/t) and initial condition (c_a).

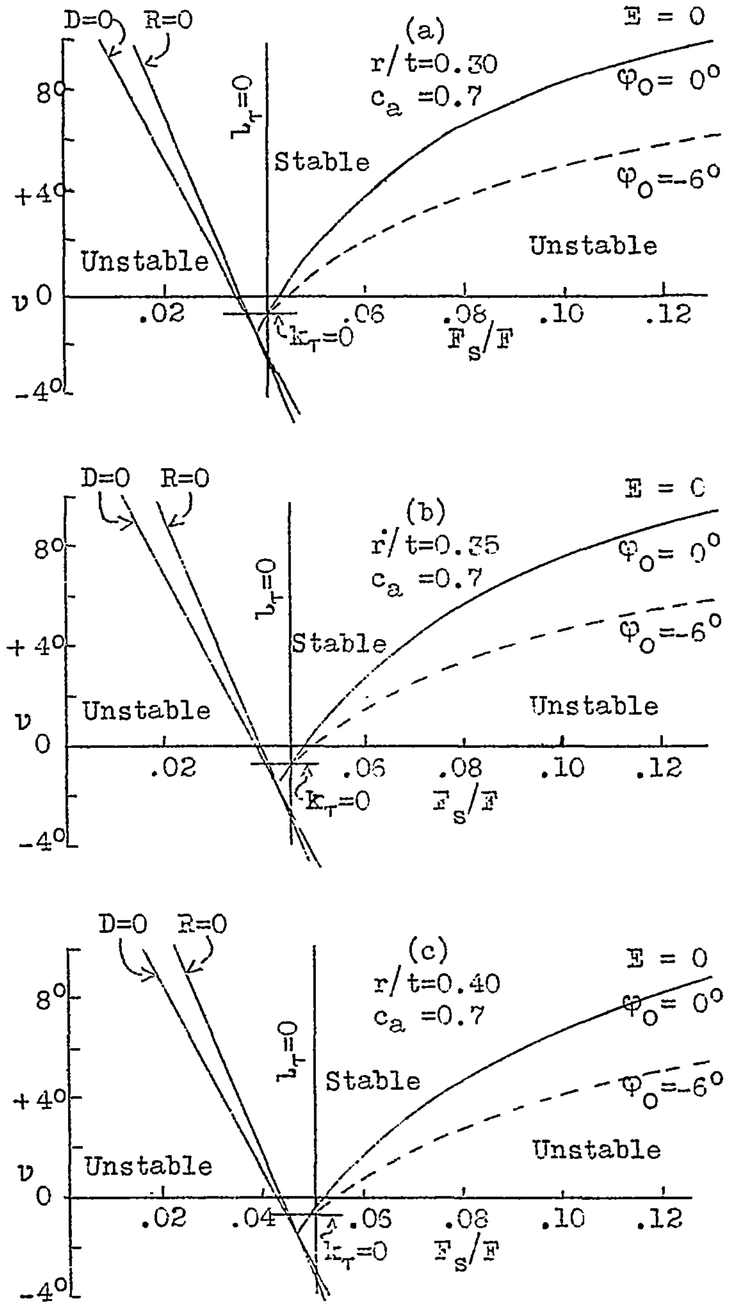

Figure 4.(example 3).-Limits of dynamic lateral stability with given initial condition (c_a)for three c.g. locations (r/t), in terms of size of vertical tail surfaces (F_s/F) and dihedral (ν).

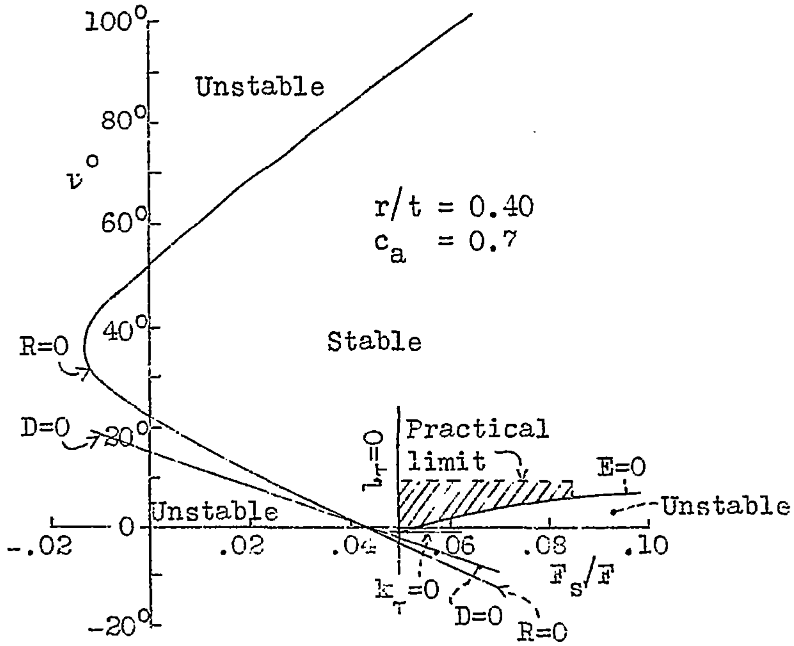

Figure 5.(example 3).-Theoretical limits of dynamic lateral stability. This figure shows in particular the course of the limit R = 0 (above ν ≈ 25° in rough approximation) and location of practical region. Fig. 4c was taken from this.

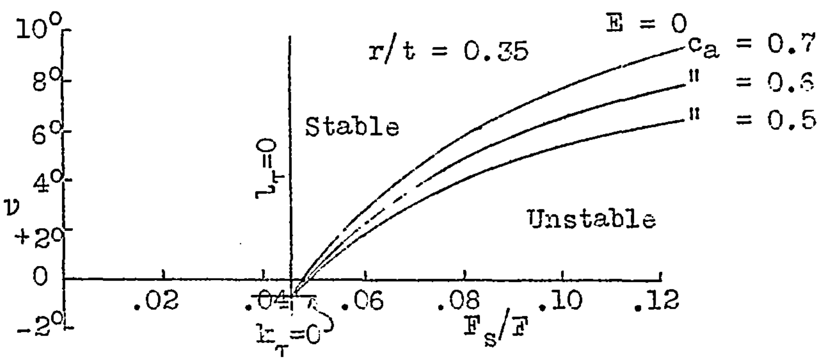

Figure 6.(example 7).-Limits of static lateral stability with given c.g. location (r/t) for three initial conditions (c_a), in terms of size of vertical tail surfaces (F_s/F) and dihedral (ν).

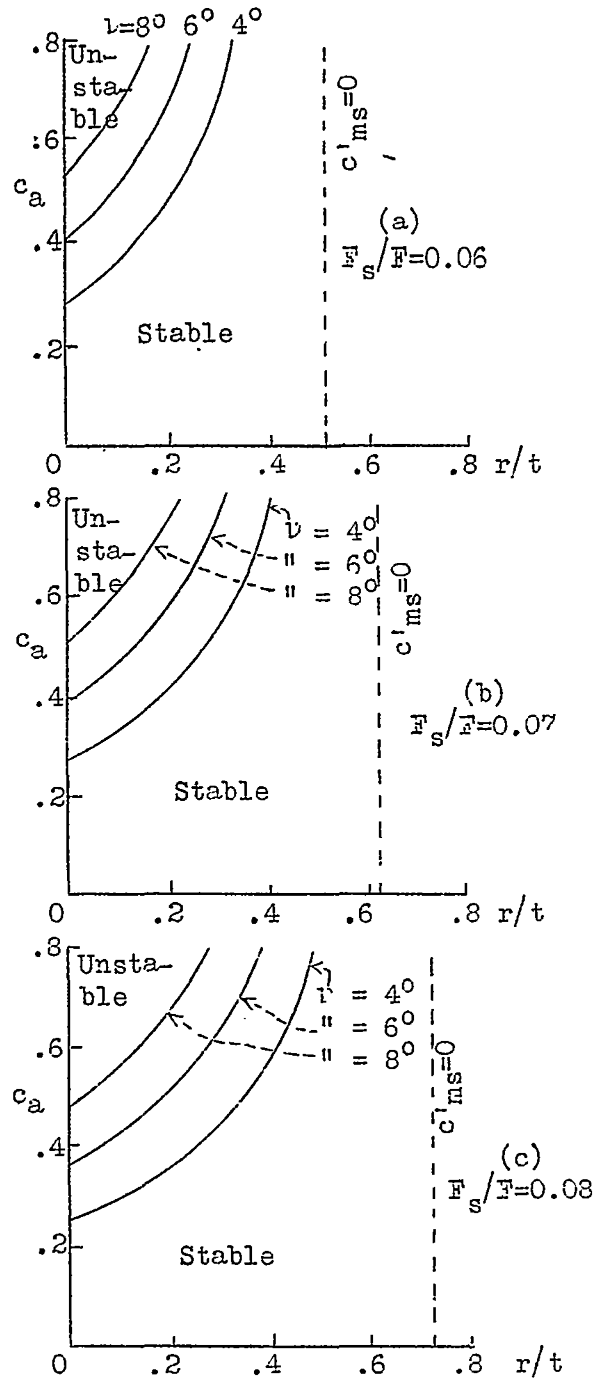

Figure 7.(example 3).-Limits of static lateral stability for three dihedral angles (ν) and three vertical-tail-surface areas (F_s/F), in terms of c.g. location and initial condition.

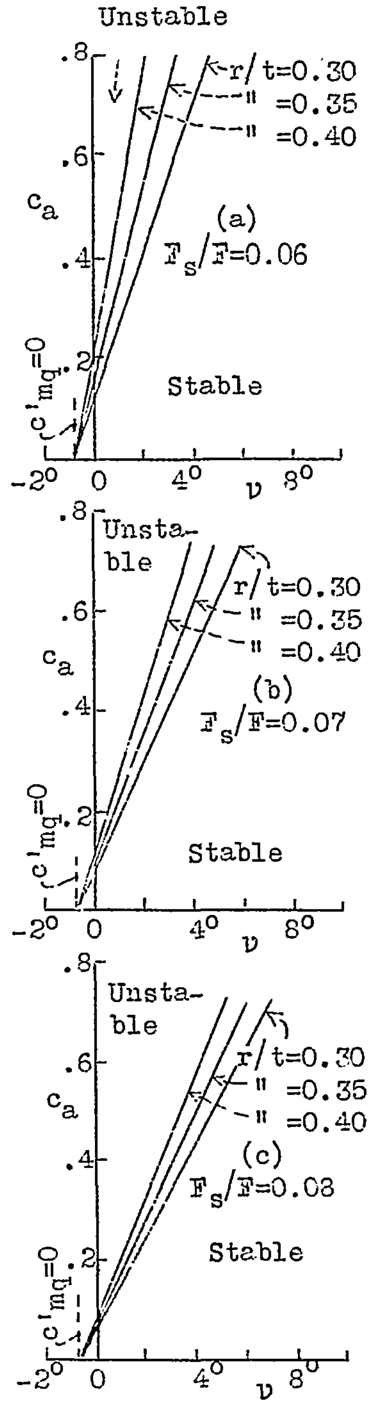

Figure 8.(example 3).-Limits of static lateral stability for three c.g. locations (r/t) and three vertical-tail-surface areas (F_s/F), in terms of dihedral (ν) and initial condition (c_a).